13

CHAPTER

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Ref: Fluid Mechanics by

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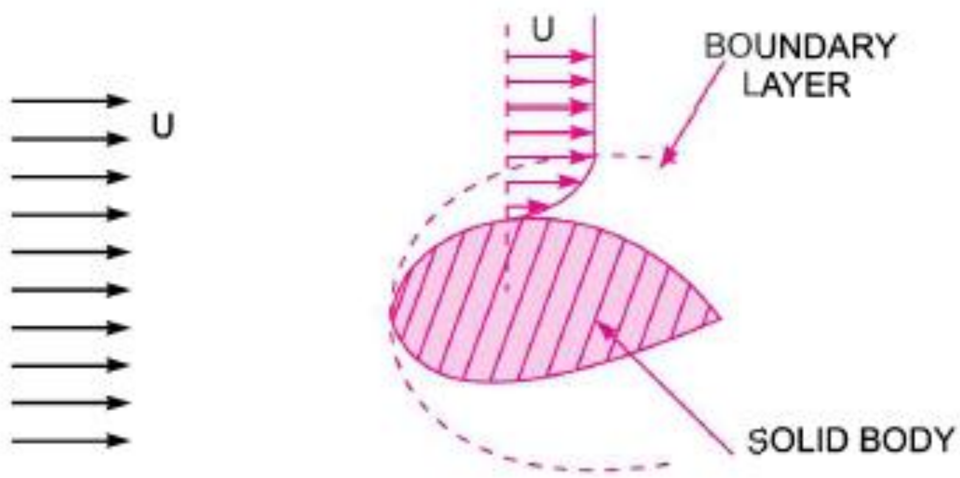
Introduction



- When the real fluid flow past solid body, the layer of the fluid adhering to the solid boundary of body and condition of no slip occurs.

- The velocity of layer of fluid adhering to the body is having velocity as that of solid boundary.

- If solid body is fixed, then velocity will be xero.

- Further away from solid boundary, the velocity keeps on raising.

- As a result of this velocity variation velocity gradient exist. This variation of velocity from zero at solid boundary to free stream velocity (U) of the fluid in direction normal to the surface takes place in narrow region of solid boundary. This region is called boundary layer.

- The theory dealing with this boundary is called boundary layer theory.

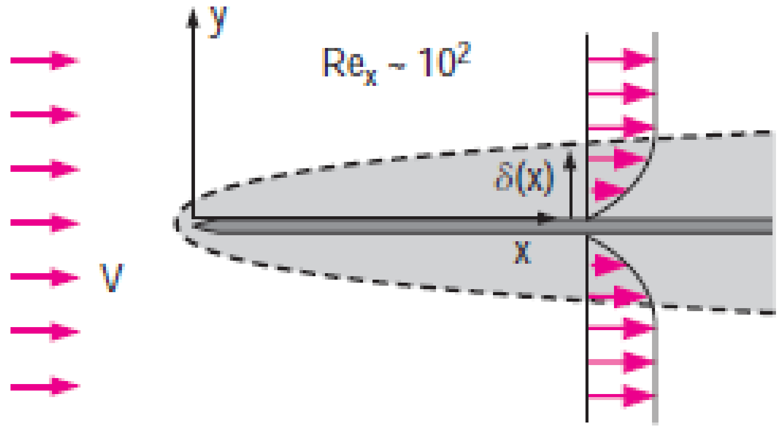
Introduction

- According to boundary layer theory, The flow of fluid neighbouring to solid boundary will be divided in two regions as shown above.

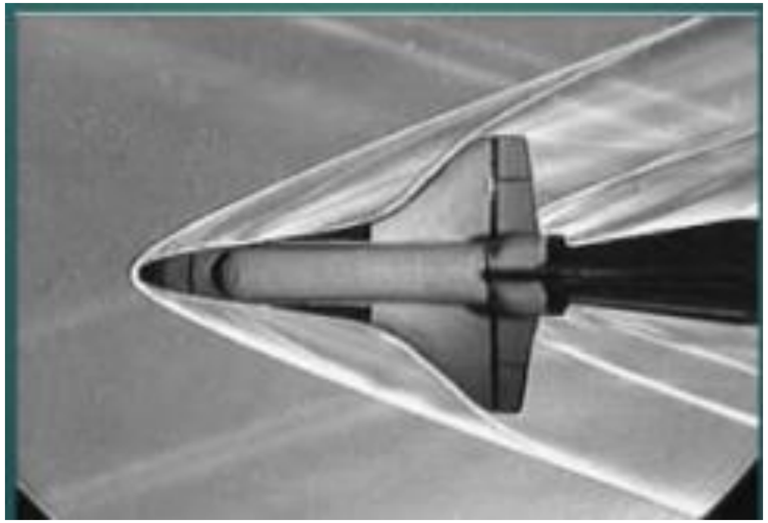
1. a very thin layer of fluid called as boundary layer in immediate neighbourhood of solid boundary where variation in velocity from zero to free stream velocity in direction normal to boundary causes velocity gradient to exist, so the shear stress over the solid boundary is

$$\tau = \mu \frac{du}{dy}$$

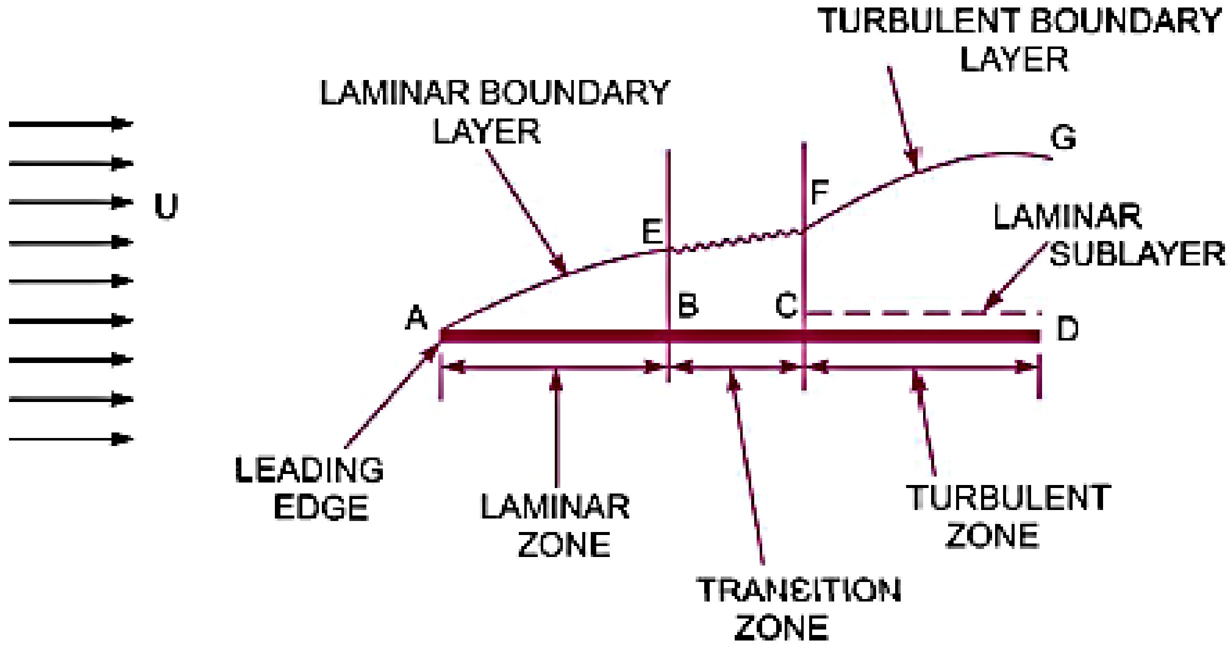
2. The remaining fluid outside the boundary layer where velocity of fluid is uniform free stream velocity. As there is no variation of velocity, velocity gradient is zero. As a result shear tress is zero.



(a)



Introduction



- **Types of boundary layer**
- Consider a flat plate having free stream velocity *U*.
- As soon as fluid moves over plate, because plate is stationary, boundary layer starts forming.

1. **Laminar boundary layer-** The leading edge of the plate where the thickness of boundary layer is small, the flow within this region is laminar, so the region where laminar flow occurs is called laminar boundary layer. And the length of plate over which laminar boundary layer exist is called laminar zone(x).

- This could be evaluated using Reynolds number for flat plate taking *(Re)x is* $5*10^{5}$*,* knowing *U* and ϑ

$$(R_e)_x = \frac{U \times x}{\nu}$$

x = Distance from leading edge,
U = Free-stream velocity of fluid,
ν = Kinematic viscosity of fluid,

Introduction

❑Turbulent boundary layer

- If the length of plate is more than x as evaluated from Reynolds number, the thickness of boundary layer will go on increasing in downstream direction.
- The laminar flow becomes unstable and transition of flow occurs from lainar to turbulent within boundary layer. This short length of plate over which transition occurs is called transition boundary layer.
- Further downstream the flow becomes turbulent and turbulent boundary layer exist over remaining length of plate.

❑Laminar sub layer

- This is the region in turbulent boundary layer zone adjuscent to the solid surface.
- In this layer even though velocity variation is parabolic, assuming very small thickness, the velocity distribution is taking to be uniform.

❑Boundary layer thickness($\boldsymbol{\delta}$)

- It is defined as distance from solid body measured in y-direction to the point where the velocity of fluid is approximately equal to 0.99 times free stream velocity *U*.
- It is denoted by symbol 'δ '
- There are three types of boundary layer thickness
- δlam- Laminar boundary layer thickness
- δtur- Turbulent boundary layer thickness
- δ' – thickness of laminar sub layer.

Introduction

- **Displacement thickness (δ^*)**

- It is the distance perpendicular to the boundary , by which free stream is displaced due to the formation of boundary layer.
- This thickness compensate for reduction in discharge/flow rate
- Mathematically,

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy.$$

- **Momentum Thickness (θ)**

- Additional thickness perpendicular to the boundary by which free stream should be displaced due to formation of boundary layer
- This thickness compensate for reduction in momentum of fluid.
- Mathematically

$$\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy.$$

Introduction

- **Energy Thickness(δ^{**})**

- \- Additional thickness perpendicular to the boundary by which free stream should be displaced due to formation of boundary layer
- This thickness compensate for reduction in energy of fluid.
- Mathematically

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy.$$

Problem 1 *Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by* $\frac{u}{U} = \frac{y}{\delta}$, *where u is the velocity at a distance y from the plate and u = U at y = δ, where δ = boundary layer thickness. Also calculate the value of δ*/θ.*

Solution. Given :

Velocity distribution $\frac{u}{U} = \frac{y}{\delta}$

(*i*) Displacement thickness δ* is given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy \qquad \left\{\because \frac{u}{U} = \frac{y}{\delta}\right\}$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta \qquad \{\delta \text{ is constant across a section}\}$$

$$= \delta - \frac{\delta^2}{2\delta} = \delta - \frac{\delta}{2} = \frac{\delta}{2}. \quad \textbf{Ans.}$$

(*ii*) Momentum thickness, θ is given by

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Substituting the value of $\frac{u}{U} = \frac{y}{\delta}$,

$$\theta = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{3\delta - 2\delta}{6} = \frac{\delta}{6}. \textbf{ Ans.}$$

(*iii*) Energy thickness δ^{**} is given by

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy = \int_0^\delta \frac{y}{\delta}\left[1 - \frac{y^2}{\delta^2}\right] dy \qquad \left\{\because \frac{u}{U} = \frac{y}{\delta}\right\}$$

$$= \int_0^\delta \left[\frac{y}{\delta} - \frac{y^3}{\delta^3}\right] dy = \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^\delta = \frac{\delta^2}{2\delta} - \frac{\delta^4}{4\delta^3}$$

$$= \frac{\delta}{2} - \frac{\delta}{4} = \frac{2\delta - \delta}{4} = \frac{\delta}{4}. \textbf{ Ans.}$$

(*iv*)

$$\frac{\delta^*}{\theta} = \frac{\left(\frac{\delta}{2}\right)}{\left(\frac{\delta}{6}\right)} = \frac{\delta}{2} \times \frac{6}{\delta} = 3. \textbf{ Ans.}$$

Problem 2 *Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by* $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$.

Solution. Given :

Velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(*i*) Displacement thickness δ^* is given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Substituting the value of $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$, we have

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \int_0^\delta \left\{1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right\} dy = \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3}. \quad \textbf{Ans.}$$

(*ii*) Momentum thickness θ, is given by

$$\theta = \int_0^\delta \frac{u}{U}\left\{1 - \frac{u}{U}\right\} dy = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy = \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$= \left[\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}\right] = \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$= \frac{15\delta - 25\delta + 15\delta - 3\delta}{15} = \frac{30\delta - 28\delta}{15} = \frac{\mathbf{2\delta}}{\mathbf{15}}.$$ **Ans.**

(*iii*) Energy thickness δ** is given by

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1-\frac{u^2}{U^2}\right]dy = \int_0^\delta \left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left(1-\left[\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right]^2\right)dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left(1-\left[\frac{4y^2}{\delta^2}+\frac{y^4}{\delta^4}-\frac{4y^3}{\delta^3}\right]\right)dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left(1-\frac{4y^2}{\delta^2}-\frac{y^4}{\delta^4}+\frac{4y^3}{\delta^3}\right)dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta}-\frac{8y^3}{\delta^3}-\frac{2y^5}{\delta^5}+\frac{8y^4}{\delta^4}-\frac{y^2}{\delta^2}+\frac{4y^4}{\delta^4}+\frac{y^6}{\delta^6}-\frac{4y^5}{\delta^5}\right)dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta}-\frac{y^2}{\delta^2}-\frac{8y^3}{\delta^3}+\frac{12y^4}{\delta^4}-\frac{6y^5}{\delta^5}+\frac{y^6}{\delta^6}\right]dy$$

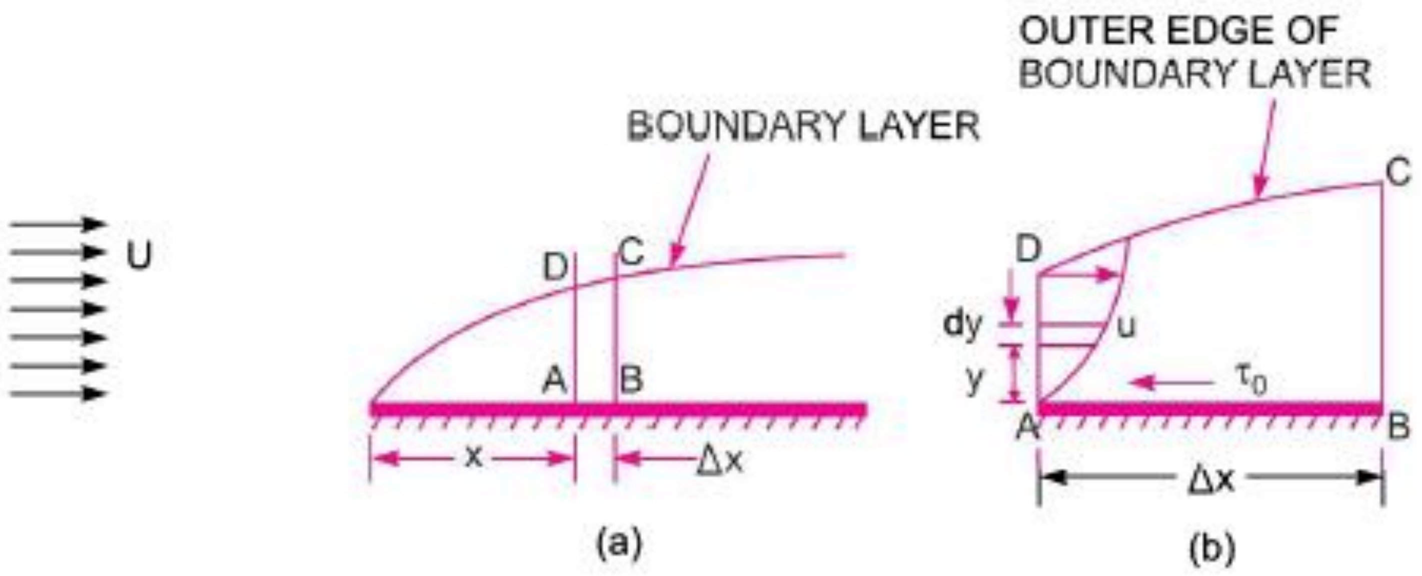
$$= \left[\frac{2y^2}{2\delta}-\frac{y^3}{3\delta^2}-\frac{8y^4}{4\delta^3}+\frac{12y^5}{5\delta^4}-\frac{6y^6}{6\delta^5}+\frac{y^7}{7\delta^6}\right]_0^\delta$$

$$= \frac{\delta^2}{\delta}-\frac{\delta^3}{3\delta^2}-\frac{2\delta^4}{\delta^3}+\frac{12\delta^5}{5\delta^4}-\frac{\delta^6}{\delta^5}+\frac{\delta^7}{7\delta^6} = \delta-\frac{\delta}{3}-2\delta+\frac{12}{5}\delta-\delta+\frac{\delta}{7}$$

$$= -2\delta-\frac{\delta}{3}+\frac{12}{5}\delta+\frac{\delta}{7} = \frac{-210\delta-35\delta+252\delta+15\delta}{105}$$

$$= \frac{-245\delta+267\delta}{105} = \mathbf{\frac{22\delta}{105}}. \quad \textbf{Ans.}$$

VON-KARMAM MOMENTUM INTEGRAL EQUATION



- consider a flow of fluid with free stream velocity 'U' over a thin plate as shown in figure.
- The drag force can be determined on the plate if velocity profile near plate is known.
- Von-Karman suggested an equation by which the drag can be calculated . The equation is

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$$

Where, τ_0- wall shear stress
ρ- density of the fluid
θ- momentum thickness
x- length of plate

This is applied to :

1. Laminar boundary layers,
2. Transition boundary layers, and
3. Turbulent boundary layer flows.

VON-KARMAM EQUATION

- For the given velocity in laminar zone, transition zone or turbulent zone of a boundary layer, the shear stress τ_0 could be obtained from above equation.
- So, the drag force on small distance Δx of the plate will be

$$\Delta F_D = \tau_0 \times \Delta x \times b$$

- So, the total drag on plate of length L is

$$F_D = \int \Delta F_D = \int_0^L \tau_0 \times b \times dx \quad \{\text{change } \Delta x = dx\}.$$

- Local Coefficient of drag $(C_D{}^*)$

$$C_D{}^* = \frac{\tau_0}{\frac{1}{2}\rho U^2}.$$

- Average coefficient of drag(C_D)

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$$

where A = Area of the surface (or plate)

U = Free-stream velocity

ρ = Mass density of fluid.

VON-KARMAM EQUATION

- **Boundary conditions for velocity profile**

- The following boundary conditions must be satisfied by the velocity profile whether is laminar or turbulent boundary layer.
- The conditions are

1. At $y = 0$, $u = 0$ and $\frac{du}{dy}$ has some finite value
2. At $y = \delta$, $u = U$
3. At $y = \delta$, $\frac{du}{dy} = 0$.

Problems On Laminar Boundary Layer Velocity Distributions

Problem 13.5 *For the velocity profile for laminar boundary layer* $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. *Determine the boundary layer thickness, shear stress, drag force and co-efficient of drag in terms of Reynold number.*

Solution. Given :

Velocity distribution, $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

Using equation (13.10), we have $\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$

Substituting the value of $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ in the above equation

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\left[1 - \left\{\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right\}\right] dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\left(1 - \frac{3y}{2\delta} + \frac{y^3}{2\delta^3}\right) dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2} + \frac{3y^4}{4\delta^4} - \frac{y^3}{2\delta^3} + \frac{3y^4}{4\delta^4} - \frac{y^6}{4\delta^6}\right) dy\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{3y^2}{2\times 2\delta} - \frac{9y^3}{3\times 4\delta^2} + \frac{3y^5}{5\times 4\delta^4} - \frac{y^4}{4\times 2\delta^3} + \frac{3y^5}{5\times 4\delta^4} - \frac{y^7}{7\times 4\delta^6}\right]_0^\delta$$

$$= \frac{\partial}{\partial x}\left[\frac{3\delta^2}{4\delta} - \frac{3\delta^3}{4\delta^2} + \frac{3}{20}\frac{\delta^5}{\delta^4} - \frac{1}{8}\frac{\delta^4}{\delta^3} + \frac{3}{20}\frac{\delta^5}{\delta^4} - \frac{1}{28}\frac{\delta^7}{\delta^6}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{3}{4}\delta - \frac{3}{4}\delta + \frac{3}{20}\delta - \frac{1}{8}\delta + \frac{3}{20}\delta - \frac{1}{28}\delta\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{6}{20}\delta - \frac{1}{8}\delta - \frac{1}{28}\delta\right] = \frac{\partial\delta}{\partial x}\left[\frac{84-35-10}{280}\right] = \frac{39}{280}\frac{\partial\delta}{\partial x}$$

$$\tau_0 = \rho U^2 \times \frac{39}{280}\frac{\partial\delta}{\partial x} = \frac{39}{280}\rho U^2\frac{\partial\delta}{\partial x} \qquad \text{...(13.20)}$$

Also the shear stress τ_0 is given by $\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$, where $u = U\left[\frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3}\right]$

$$\therefore \qquad \frac{du}{dy} = U\left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right]$$

Hence $$\left(\frac{du}{dy}\right)_{y=0} = U\left[\frac{3}{2\delta} - \frac{3}{2\delta^3}\times 0\right] = \frac{3U}{2\delta}$$

$$\therefore \qquad \tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = \mu \frac{3U}{2\delta} = \frac{3}{2}\frac{\mu U}{\delta} \qquad \text{...(13.21)}$$

Equating the two values of τ_0 given by equations (13.20) and (13.21)

$$\frac{39}{280}\rho U^2 \frac{\partial \delta}{\partial x} = \frac{3}{2}\frac{\mu U}{\delta}$$

$$\therefore \qquad \delta \partial \delta = \frac{3}{2}\mu U \times \frac{280}{39} \times \frac{1}{\rho U^2}\partial x = \frac{420}{39}\frac{\mu}{\rho U}\partial x$$

Integrating, we get $\qquad \dfrac{\delta^2}{2} = \dfrac{420}{39}\dfrac{\mu}{\rho U} x + C$

where $x = 0, \delta = 0, \therefore C = 0$

$$\therefore \qquad \frac{\delta^2}{2} = \frac{420}{39} \cdot \frac{\mu}{\rho U} x$$

or

$$\delta = \sqrt{\frac{420 \times 2}{39}\frac{\mu}{\rho U} x} = 4.64\sqrt{\frac{\mu x}{\rho U}} = 4.64\sqrt{\frac{\mu x \times x}{\rho U x}}$$

$$= 4.64\sqrt{\frac{\mu}{\rho U x}}\, x = \frac{4.64\, x}{\sqrt{R_{e_x}}} \qquad \text{...(13.22)}$$

(*i*) **Shear Stress τ_0.** Substituting the value of δ from equation (13.22) into equation (13.21), we get

$$\tau_0 = \frac{3}{2}\frac{\mu U}{\dfrac{4.64\, x}{\sqrt{R_{e_x}}}} = \frac{3}{9.28}\frac{\mu U \sqrt{R_{e_x}}}{x} = 0.323\frac{\mu U}{x}\sqrt{R_{e_x}}$$

(*ii*) **Drag force (F_D)**

Using equation (13.12), we get the drag force as

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^L 0.323 \frac{\mu U}{x} \sqrt{R_{e_x}} \times b \times dx$$

$$= 0.323 \int_0^L \frac{\mu U}{x} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx = 0.323\, \mu U \sqrt{\frac{\rho U}{\mu}} \times b \int_0^L \frac{1}{\sqrt{x}} dx$$

$$= 0.323\, \mu U \sqrt{\frac{\rho U}{\mu}} \times b \int_0^L x^{-1/2}\, dx$$

$$= 0.323\, \mu U \sqrt{\frac{\rho U}{\mu}} \times b \left[\frac{x^{1/2}}{\frac{1}{2}} \right]_0^L = 0.323 \times 2\, \mu U \sqrt{\frac{\rho U}{\mu}} \times b \left[\sqrt{L} \right]$$

$$= 0.646\, \mu U \sqrt{\frac{\rho U L}{\mu}} \times b \qquad \text{...(13.23)}$$

(*iii*) **Drag Co-efficient (C_D).** Using equation (13.14), we get the value of C_D as

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}, \text{ where } A = b \times L$$

$$= \frac{0.646\,\mu U \sqrt{\frac{\rho U L}{\mu}} \times b}{\frac{1}{2}\rho \times b \times L \times U^2} = 0.646 \times 2 \times \frac{\mu}{\rho U L} \times \sqrt{\frac{\rho U L}{\mu}} = \frac{1.292}{\sqrt{\frac{\rho U L}{\mu}}}$$

$$= \frac{1.292}{\sqrt{R_{e_L}}}. \qquad \left\{\because \sqrt{\frac{\rho U L}{\mu}} = \sqrt{R_{e_L}}\right\} \qquad \text{...(13.24)}$$

Problem 13.7 *For the velocity profile for laminar boundary flow* $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$. *Obtain an expression for boundary layer thickness, shear stress, drag force on one side of the plate and co-efficient of drag in terms of Reynold number.*

Solution. (*i*) The velocity profile is $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$.

Substituting this value in equation (13.10), we have

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy\right] = \frac{\partial}{\partial x}\left[\int_0^\delta \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left[1-\sin\left(\frac{\pi}{2}.\frac{y}{\delta}\right)\right]dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]dy\right]$$

$$= \frac{\partial}{\partial x}\left[\left[\frac{-\cos\frac{\pi y}{2\delta}}{\frac{\pi}{2\delta}}\right] - \left[\frac{\frac{\pi y}{2\delta}\times\frac{1}{2}}{\frac{\pi}{2\delta}} - \frac{\sin 2\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{4\times\frac{\pi}{2\delta}}\right]\right]_0^\delta$$

$$\left\{\because \quad \int \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right) dy = \frac{\frac{\pi y}{2\delta}\times\frac{1}{2}}{\frac{\pi}{2\delta}} - \frac{\sin 2\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{4\times\frac{\pi}{2\delta}}\right\}$$

$$\because \qquad \frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\left(\frac{-\cos\frac{\pi}{2}\frac{\delta}{\delta}}{\frac{\pi}{2\delta}} + \frac{\cos\frac{\pi}{2}\times\frac{0}{\delta}}{\frac{\pi}{2\delta}}\right) - \left[\frac{\frac{\pi}{2}\frac{\delta}{\delta}\times\frac{1}{2}}{\frac{\pi}{2\delta}} - 0\right]\right]$$

$$= \frac{\partial}{\partial x}\left[\left(0 + \frac{1}{\frac{\pi}{2\delta}}\right) - \frac{\left(\frac{\pi}{4}\right)}{\frac{\pi}{2\delta}}\right] = \frac{\partial}{\partial x}\left[\frac{2\delta}{\pi} - \frac{\pi}{4}\times\frac{2\delta}{\pi}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{2\delta}{\pi} - \frac{\delta}{2}\right] = \frac{\partial}{\partial x}\left[\frac{4-\pi}{2\pi}\right]\delta = \left(\frac{4-\pi}{2\pi}\right)\frac{\partial\delta}{\partial x}$$

$$\therefore \qquad \tau_0 = \left(\frac{4-\pi}{2\pi}\right)\rho U^2 \frac{\partial\delta}{\partial x} \qquad \text{...(13.30)}$$

τ_0 is also equal $= \mu \left(\frac{du}{dy}\right)_{\text{at } y=0}$

But $$u = U \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$\therefore$ $$\left(\frac{du}{dy}\right) = U \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) \times \frac{\pi}{2\delta}$$

$$\left(\frac{du}{dy}\right)_{y=0} = U \times \frac{\pi}{2\delta} \cos\left(\frac{\pi}{2} \times \frac{0}{\delta}\right) = \frac{U\pi}{2\delta}$$

$\therefore$ $$\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U \pi}{2\delta} \qquad \text{...(13.31)}$$

Equating the two values τ_0 given by equations (13.30) and (13.31)

$$\left(\frac{4-\pi}{2\pi}\right)\rho U^2 \frac{\partial \delta}{\partial x} = \frac{\mu U \pi}{2\delta} \quad \text{or} \quad \delta \partial \delta = \frac{\mu U \pi}{2} \times \frac{2\pi}{4-\pi} \times \frac{1}{\rho U^2}\, \partial x$$

$\therefore$ $$\delta \partial \delta = \frac{\pi^2}{(4-\pi)} \frac{\mu U}{\rho U^2} \,.\, \partial x = 11.4975 \frac{\mu}{\rho U} \partial x$$

Integrating, we get $$\frac{\delta^2}{2} = 11.4975 \frac{\mu}{\rho U} x + C$$

At $x = 0$, $\delta = 0$ and hence $C = 0$

$$\therefore \qquad \frac{\delta^2}{2} = 11.4975 \frac{\mu}{\rho U} x$$

$$\therefore \qquad \delta = \sqrt{2 \times 11.4975 \frac{\mu}{\rho U} x} = 4.795 \sqrt{\frac{\mu}{\rho U}\, x}$$

$$= 4.795 \sqrt{\frac{\mu}{\rho U x}} = 4.795 \sqrt{\frac{\mu}{\rho U x}} \times x$$

$$= \frac{4.795\, x}{\sqrt{R_{e_x}}} \qquad \text{...(13.32)}$$

(*ii*) **Shear Stress (τ_0)**

From equation (13.31), $$\tau_0 = \frac{\mu U \pi}{2\delta} = \frac{\mu U \pi}{\dfrac{2 \times 4.795\, x}{\sqrt{R_{e_x}}}} = \frac{\mu U \pi \sqrt{R_{e_x}}}{2 \times 4.795\, x}$$

$$= \frac{\pi}{2 \times 4.795} \frac{\mu U}{x} \sqrt{R_{e_x}} = 0.327 \frac{\mu U}{x} \sqrt{R_{e_x}}\,.$$

(*iii*) **Drag force (F_D)** on one side of the plate is given by equation (13.12)

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^L 0.327 \frac{\mu U}{x} \sqrt{R_{e_x}} \times b \times dx = 0.327\ \mu U \times b \int_0^L \frac{1}{x} \sqrt{\frac{\rho U x}{\mu}}\, dx$$

$$= 0.327\ \mu U \times b \times \sqrt{\frac{\rho U}{\mu}} \int_0^L x^{-1/2}\, dx = 0.327\ \mu U \times b \times \sqrt{\frac{\rho U}{\mu}} \left[\frac{x^{1/2}}{\frac{1}{2}}\right]_0^L$$

$$= 0.327 \times 2 \times \mu U \times b \sqrt{\frac{\rho U}{\mu}} \times \sqrt{L}$$

$$= 0.655 \times \mu U \times b \times \sqrt{\frac{\rho U L}{\mu}} \qquad \text{...(13.33)}$$

(*iv*) **Co-efficient of drag, C_D** is given by equation (13.14),

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U^2}, \text{ where } A = b \times L$$

$$\therefore \qquad C_D = \frac{0.655 \times \mu U \times b \times \sqrt{\frac{\rho U L}{\mu}}}{\frac{1}{2} \rho U^2 \times b \times L} = 0.655 \times 2 \times \frac{\mu}{\rho U L} \times \sqrt{\frac{\rho U L}{\mu}}$$

$$= 1.31 \times \frac{1}{\sqrt{\frac{\rho U L}{\mu}}} = \frac{1.31}{\sqrt{R_{e_L}}} \qquad \text{...(13.34)}$$

Note. $\int \sin^2 x ds = \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)$ is used.

LAMINAR BOUNDARY LAYER VELOCITY DISTRIBUTIONS

	Velocity Distribution	δ	C_D
1.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$5.48\ x/\sqrt{R_{e_x}}$	$1.46/\sqrt{R_{e_L}}$
2.	$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$4.64\ x/\sqrt{R_{e_x}}$	$1.292/\sqrt{R_{e_L}}$
3.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$	$5.84\ x/\sqrt{R_{e_x}}$	$1.36/\sqrt{R_{e_L}}$
4.	$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$	$4.79\ x/\sqrt{R_{e_x}}$	$1.31/\sqrt{R_{e_L}}$
5.	Blasius's Solution	$4.91\ x/\sqrt{R_{e_x}}$	$1.328/\sqrt{R_{e_L}}$

TURBULENT BOUNDARY LAYER VELOCITY DISTRIBUTIONS

- For the turbulent boundary layer on the flat plat pate, the important parameters are
  - The thickness of boundary layer
  - Drag force on one side of flat plate
  - Coefficient of drag
- The velocity profile due to turbulent boundary layer with zero pressure gradient is given by Blasius

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^n$$

where $n = \frac{1}{7}$ for $R_e < 10^7$ but more than 5×10^5

$$\therefore \qquad \frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

- The value for shear stress τ_0 is taken as $\tau_0 = 0.0225\, \rho\, U^2 \left(\frac{\mu}{\rho \delta U}\right)^{1/4}$

TURBULENT BOUNDARY LAYER VELOCITY DISTRIBUTIONS

Problem 13.13 *For the velocity profile for turbulent boundary layer* $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$*, obtain an expression for boundary layer thickness, shear stress, drag force on one side of the plate and co-efficient of drag in terms of Reynold number. Given the shear stress (*τ_0*) for turbulent boundary layer as*

$$\tau_0 = 0.0225 \, \rho \, U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}.$$

Solution. Given : $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$

(*i*) Substituting this value in Von Karman momentum integral equation (13.10),

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy\right] = \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{y^{1/7}}{\delta^{1/7}} - \frac{y^{2/7}}{\delta^{2/7}}\right) dy\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{y^{1/7+1}}{\left(\frac{1}{7}+1\right)\delta^{1/7}} - \frac{y^{2/7+1}}{\left(\frac{2}{7}+1\right)\delta^{2/7}}\right]_0^\delta$$

$$= \frac{\partial}{\partial x}\left[\frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{y^{9/7}}{\delta^{2/7}}\right]_0^\delta = \frac{\partial}{\partial x}\left[\frac{7}{8}\frac{\delta^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{\delta^{9/7}}{\delta^{2/7}}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{7}{8}\delta - \frac{7}{9}\delta\right] = \frac{\partial}{\partial x}\left[\frac{63-56}{72}\right]\delta = \frac{\partial}{\partial x}\left[\frac{7}{72}\right]\delta = \frac{7}{72}\frac{\partial \delta}{\partial x}$$

In the above expression, the integration limits should be from δ' to δ. But as the laminar sub-layer is very thin that is δ' is very small. Hence the limits of integration are taken from 0 to δ.

Now $$\tau_0 = \frac{7}{72}\rho U^2 \frac{\partial \delta}{\partial x} \quad \text{...(13.38)}$$

But the value of τ_0 for turbulent boundary layer is given,

$$\tau_0 = 0.0225\, \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4} \quad \text{...(13.39)}$$

Equating the two values of τ_0 given by equations (13.38) and (13.39), we have

$$\frac{7}{72}\rho U^2 \frac{\partial \delta}{\partial x} = .0225\, \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$$

or $$\frac{7}{72}\frac{\partial \delta}{\partial x} = .0225 \left(\frac{\mu}{\rho U}\right)^{1/4} \times \frac{1}{\delta^{1/4}} \quad \{\text{cancelling } \rho U^2\}$$

or $$\delta^{1/4}\partial\delta = .0225 \times \frac{72}{7} \times \left(\frac{\mu}{\rho U}\right)^{1/4} \partial x = 0.2314 \left(\frac{\mu}{\rho U}\right)^{1/4} \partial x.$$

Integrating, we get $$\frac{\delta^{1/4+1}}{\left(\frac{1}{4}+1\right)} = 0.2314 \left(\frac{\mu}{\rho U}\right)^{1/4} x + C$$

or $$\frac{4}{5} \times \delta^{5/4} = 0.2314 \left(\frac{\mu}{\rho U}\right)^{1/4} x + C$$

where C is constant of integration.

To determine the value of C, assume turbulent boundary layer starts from the leading edge, though in actual practice the turbulent boundary layer starts after the transition from laminar boundary layer. The laminar layer exists for a very short distance and hence this assumption will not affect the subsequent analysis.

Hence at $x = 0$, $\delta = 0$ and so $C = 0$

$$\therefore \quad \frac{4}{5}\delta^{5/4} = 0.2314\left(\frac{\mu}{\rho U}\right)^{1/4} x \text{ or } \delta^{5/4} = \frac{0.2314 \times 5}{4}\left(\frac{\mu}{\rho U}\right)^{1/4} x$$

or
$$\delta = \left[\frac{0.2314 \times 5}{4}\left(\frac{\mu}{\rho U}\right)^{1/4} x\right]^{4/5} = \left(\frac{0.2314 \times 5}{4}\right)^{4/5}\left(\frac{\mu}{\rho U}\right)^{1/5} x^{4/5}$$

$$= 0.37\left(\frac{\mu}{\rho U}\right)^{1/5} x^{4/5} \qquad \text{...(13.40)}$$

$$= 0.37\left(\frac{\mu}{\rho U x}\right)^{1/5} x^{1/5} \times x^{4/5} = 0.37\left(\frac{1}{R_{e_x}}\right)^{1/5} \times x = \frac{0.37x}{\left(R_{e_x}\right)^{1/5}} \qquad \text{...(13.41)}$$

From equation (13.40), it is clear that δ varies as $x^{4/5}$ in turbulent boundary layer while in case of laminar boundary layer δ varies as $\sqrt{x}$.

(*ii*) **Shear Stress (τ_0)** at any point from leading edge is given by equation (13.40) as

$$\tau_0 = 0.225\ \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$$

Substituting the value of δ from equation (13.40), we have

$$\tau_0 = 0.0225\ \rho U^2 \left(\frac{\mu}{\rho U \times 0.37 \times \left(\frac{\mu}{\rho U}\right)^{1/5} \times x^{4/5}}\right)^{1/4}$$

$$= \frac{.0225 \times 2}{2} \rho U^2 \left(\frac{\mu^{4/5}}{0.37 \times (\rho U)^{4/5} \times x^{4/5}}\right)^{1/4}$$

$$= .0225 \times 2 \times \frac{\rho U^2}{2} \times \frac{1}{(0.37)^{1/4}} \left(\frac{\mu}{\rho U x}\right)^{1/5}$$

$$= 0.0577 \times \frac{\rho U^2}{2} \left(\frac{\mu}{\rho U x}\right)^{1/5} \qquad \text{...(13.42)}$$

(*iii*) **Drag force (F_D) on one side of the plate is**

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^L 0.0577 \times \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/5} \frac{1}{x^{1/5}} \times b \times dx$$

$$= 0.0577 \times \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/5} \times b \int_0^L x^{-1/5}\, dx$$

$$= .0577 \times \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/5} \times b \times \left[\frac{x^{4/5}}{4/5}\right]_0^L$$

$$= .0577 \times \frac{5}{4} \times \frac{\rho U^2}{2} \left(\frac{\mu}{\rho U}\right)^{1/5} \times b \times L^{4/5}$$

$$= 0.072 \times \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/5} \times b \times L^{4/5}$$

(*iv*) **Drag co-efficient, C_D is given by**

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}, \text{ where } A = L \times b$$

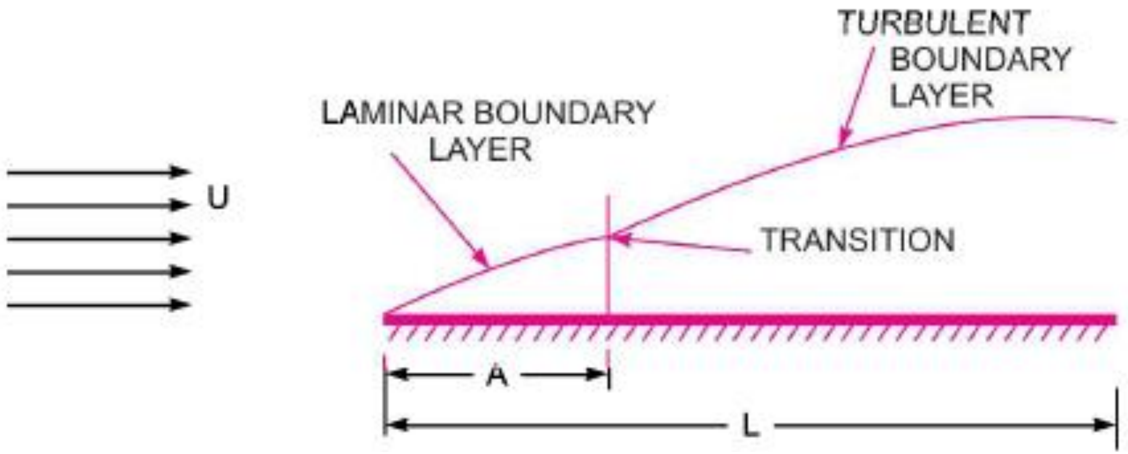
$$= \frac{.072 \times \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/5} \times b \times L^{4/5}}{\frac{\rho U^2}{2} \times b \times L}$$

$$= 0.072 \times \left(\frac{\mu}{\rho U}\right)^{1/5} \cdot \frac{1}{L^{1/5}} = 0.072 \left(\frac{\mu}{\rho U L}\right)^{1/5}$$

$$= \frac{.072}{R_{e_L}^{1/5}} \quad \text{...(13.43)} \quad \left\{\because \quad R_{e_L} = \frac{\rho U L}{\mu}\right\}$$

This is valid for $R_{e_L} > 5 \times 10^5$ but less than 10^7.

Total Drag On Flat Plate Due To Laminar And Turbulent Boundary Layer



Let L = Total length of the plate, b = Width of plate,

A = Length of laminar boundary layer

If the length of transition region is assumed negligible, then

$L - A$ = Length of turbulent boundary layer.

(1) Find the length from the leading edge upto which laminar boundary layer exists. This is done by equating $5 \times 10^5 = \frac{Ux}{\nu}$. The value of x gives the length of laminar boundary layer. Let this length is equal to A.

(2) Find drag using Blasius solution for laminar boundary layer for length A.

(3) Find drag due to turbulent boundary layer for the whole length of the plate.

(4) Find the drag due to turbulent boundary layer for a length A only

Then total drag on the plate

= Drag given by (2) + Drag given by (3) – Drag given by (4)

= Drag due to laminar boundary layer for length A

+ Drag due to turbulent boundary layer for length L

– Drag due to turbulent boundary layer for length A. ...(13.45)

Problem 13.16 (A) *Air flows at 10 m/s past a smooth rectangular flat plate 0.3 m wide and 3 m long. Assuming that the turbulence level in the oncoming stream is low and that transition ocurs at* $R_e = 5 \times 10^5$*, calculate ratio of total drag when the flow is parallel to the length of the plate to the value when the flow is parallel to the width.* (R.G.P.V., Bhopal S 2001)

Solution. Given :

$$U = 10 \text{ m/s} \; ; \; b = 0.3 \text{ m} \; ; \; L = 3 \text{ m} \; ;$$

Reynolds number for laminar B.L. $= 5 \times 10^5$.

The kinematic viscosity of air and density of air may be assumed as their values are not given in the question. Take $\rho = 1.24 \text{ kg/m}^2$ and $\nu = 0.15$ stoke

$$\therefore \qquad \rho = 1.24 \text{ kg/m}^3 \text{ and } \nu = 0.15 \text{ stoke} = 0.15 \times 10^{-4} \text{ m}^2\text{/s}.$$

(*i*) **Drag when flow is parallel to the length of the plate**

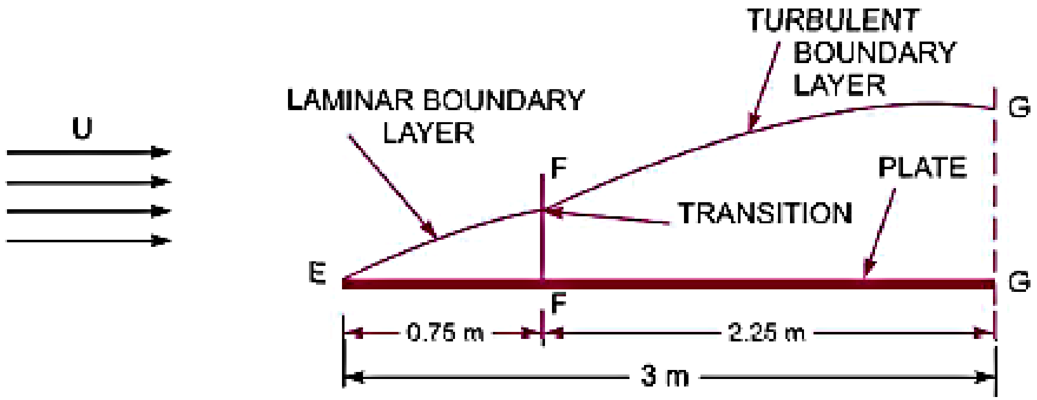
Let x = the distance from leading edge upto which laminar boundary exists

$$\therefore \qquad 5 \times 10^5 = \frac{\rho \times U \times x}{\mu} = \frac{U \times x}{\nu} = \frac{10 \times x}{0.15 \times 10^{-4}}$$

$$\therefore \qquad x = \frac{5 \times 10^5 \times 0.15 \times 10^{-4}}{10} = 0.75 \text{ m}$$

Now the drag force on the plate on one side

= Drag due to laminar boundary layer + Drag due to turbulent boundary layer ...(*i*)



(*a*) Drag due to laminar boundary layer (*i.e.*, from E to F)

$$F_{EF} = \frac{1}{2}\,\rho A U^2 \times C_D$$

'here C_D is given by Blasius solution for laminar boundary layer as

$$C_D = \frac{1.328}{\sqrt{R_{e_x}}}, \text{ where } R_{ex} = 5 \times 10^5$$

$$= \frac{1.328}{\sqrt{5 \times 10^5}} = 0.001878$$

A = Area of plate upto laminar boundary layer

$= 0.75 \times b = 0.75 \times 0.3 = 0.225 \text{ m}^2$

$\rho = 1.24 \text{ kg/m}^3$

$$\therefore \quad F_{EF} = \frac{1}{2} \times 1.24 \times 0.225 \times 10^2 \times 0.001878 = 0.0262 \text{ N}$$

(*b*) **Drag force due to turbulent boundary layer from F to G**

= Drag force due to turbulent boundary layer from E to G

– Drag force due to turbulent B.L. from E to F

$$= (F_{EG})_{\text{turb.}} - (F_{EF})_{\text{turb.}}$$

Now $$(F_{EG})_{\text{turb.}} = \frac{1}{2}\rho A U^2 \times C_D$$

where C_D for turbulent boundary layer is given by equation (13.44) as

$$C_D = \frac{0.072}{\left(R_{e_L}\right)^{1/5}}$$

But $$R_{e_L} = \frac{U \times L}{\nu} = \frac{10 \times 3}{0.15 \times 10^{-4}} = 20 \times 10^5$$

$\therefore$ $$C_D = \frac{0.072}{\left(20 \times 10^5\right)^{1/5}} = 0.00395$$

$\therefore$ $$(F_{EG})_{\text{turb.}} = \frac{1}{2}\rho A U^2 \times C_D = \frac{1}{2} \times 1.24 \times (3 \times 0.3) \times 10^2 \times 0.00395$$

$$= \mathbf{0.2204\ N}$$

Now $$(F_{EF})_{\text{turb.}} = \frac{1}{2}\rho \times A_{EF} \times U^2 \times C_D$$

where A_{EF} = Area of plate upto $EF = EF \times b = 0.75 \times 0.3 = 0.225 \text{ m}^2$

and $$C_D = \frac{0.072}{\left[(R_e)_{EF}\right]^{1/5}} = \frac{0.072}{\left(5 \times 10^5\right)^{1/5}} = 0.00522$$

∴ $$(F_{EF})_{\text{turb.}} = \frac{1}{2} \times 1.24 \times 0.225 \times 10^2 \times 0.00522 = \mathbf{0.0728\ N}$$

∴ Drag force due to turbulent boundary layer from F to G

$= (F_{EG})_{\text{turb.}} - (F_{EF})_{\text{turb.}} = 0.2204 - 0.0728 =$ **0.1476 N**

∴ Total drag force when flow is parallel to the length of the plate

= Drag due to laminar boundary layer upto F

+ Drag due to turbulent boundary layer from F to G

$= 0.0262 + 0.1476 =$ **0.1738 N** ...(A)

We have already calculated that upto the length of 0.75 m from the leading edge, the boundary layer is laminar. As the width of the plate is only 0.3 m, hence when flow is parallel to the width of the plate, only laminar boundary layer will be formed.

$\therefore$ Drag force on the plate

$$= \frac{1}{2} \rho A U^2 \times C_D$$

where C_D from Blasius solution for laminar boundary layer is given as

$$C_D = \frac{1.328}{\sqrt{R_{e_x}}}, \text{ here } \quad x = \text{width of plate} = 0.3 \text{ m hence}$$

$$R_{e_x} = \frac{U \times x}{\nu} = \frac{10 \times 0.3}{0.15 \times 10^{-4}} = 2 \times 10^5$$

$$= \frac{1.328}{\sqrt{2 \times 10^5}} = 0.00297$$

A = Area of plate upto width (0.3 m) = $3 \times 0.3 = 0.9$

$\rho = 1.24 \text{ kg/m}^3$

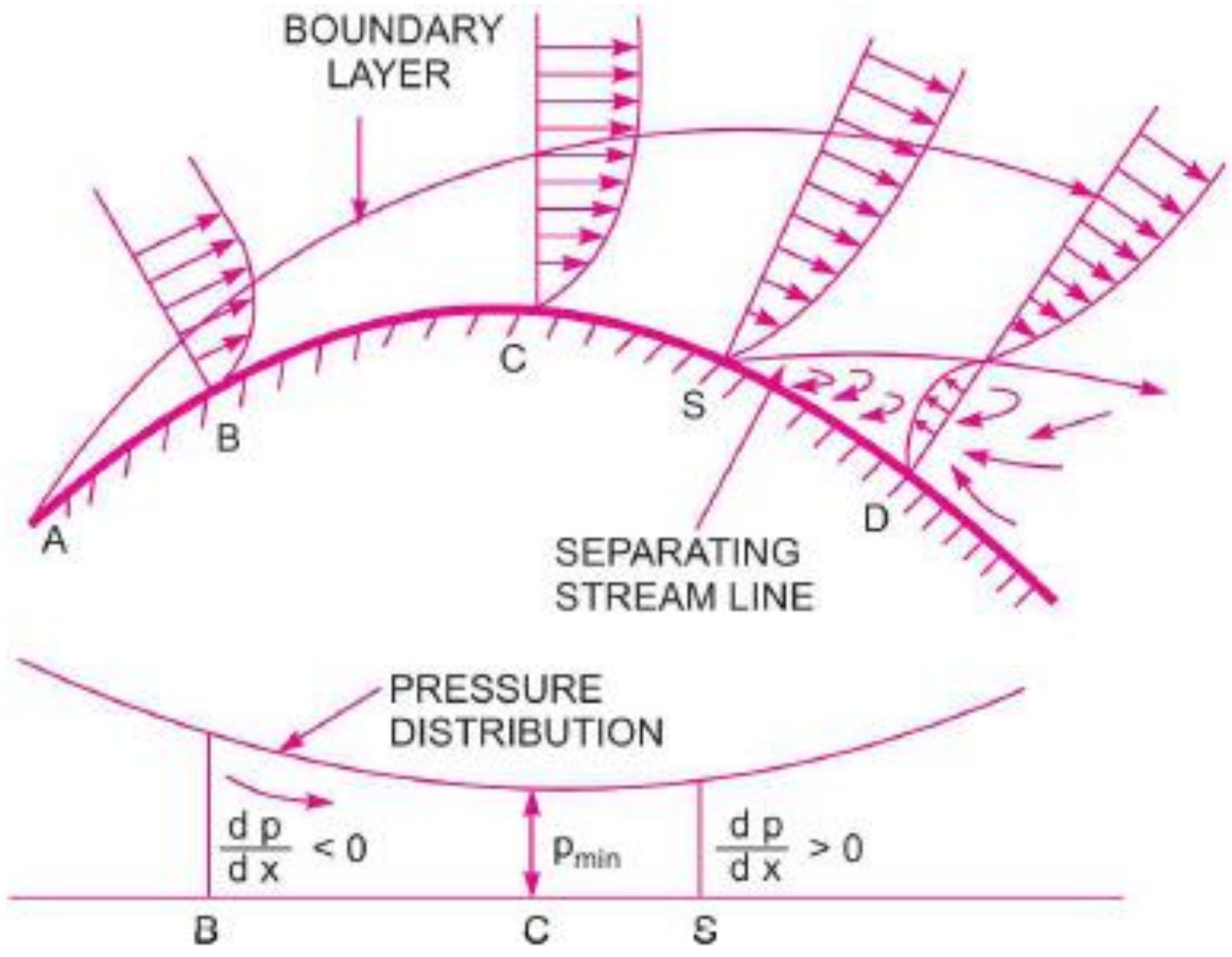
$\therefore$ Total drag on the plate $= \frac{1}{2} \times 1.24 \times 0.9 \times 10^2 \times 0.00297$

$=$ **0.1657 N** ...(*B*)

$\therefore$ Ratio of two total drags given by equations (*A*) and (*B*) becomes as

$$\frac{\text{Total drag when flow is parallel to the length of the plate}}{\text{Total drag when flow is parallel to the width of the plate}} = \frac{\text{Equation }(A)}{\text{Equation }(B)} = \frac{0.1738}{0.1657} = \mathbf{1.05.\ Ans.}$$

Separation of Boundary Layer



1. If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ Is negative---- the flow is already separated

2. If $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$ ---- the flow is on the verge of separation

3. If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ Is positive---- the flow will not separate at all or the flow remains attached with the flow

Problem 13.18 *For the following velocity profiles, determine whether the flow has separated or on the verge of separation or will attach with the surface :*

(*i*) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$, (*ii*) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3$,

(*iii*) $\frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2$.

Solution. Given :

1st Velocity Profile

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \quad \text{or} \quad u = \frac{3U}{2}\left(\frac{y}{\delta}\right) - \frac{U}{2}\left(\frac{y}{\delta}\right)^3$$

Differentiating w.r.t. y, the above equation becomes,

$$\frac{\partial u}{\partial y} = \frac{3U}{2} \times \frac{1}{\delta} - \frac{U}{2} \times 3\left(\frac{y}{\delta}\right)^2 \times \frac{1}{\delta}$$

At $y = 0$, $\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{3U}{2\delta} - \frac{3U}{2}\left(\frac{0}{\delta}\right)^2 \times \frac{1}{\delta} = \frac{3U}{2\delta}$.

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is positive. Hence flow will not separate or flow will remain attached with the surface.

2nd Velocity Profile

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3$$

$\therefore$
$$u = 2U\left(\frac{y}{\delta}\right)^2 - U\left(\frac{y}{\delta}\right)^3$$

$\therefore$
$$\frac{\partial u}{\partial y} = 2U \times 2\left(\frac{y}{\delta}\right) \times \frac{1}{\delta} - U \times 3\left(\frac{y}{\delta}\right)^2 \times \frac{1}{\delta}$$

at $y = 0, \left(\frac{\partial u}{\partial y}\right)_{y=0} = 2U \times 2\left(\frac{0}{\delta}\right) \times \frac{1}{\delta} - U \times 3\left(\frac{0}{\delta}\right)^2 \times \frac{1}{\delta} = 0$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$, the flow is on the verge of separation. **Ans.**

3rd Velocity Profile

$$\frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2$$

$$\therefore \quad u = -2U\left(\frac{y}{\delta}\right) + U\left(\frac{y}{\delta}\right)^2$$

$$\therefore \quad \frac{\partial u}{\partial y} = -2U\left(\frac{1}{\delta}\right) + 2U\left(\frac{y}{\delta}\right) \times \frac{1}{\delta}$$

at $y = 0, \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{2U}{\delta} + 2U\left(\frac{0}{\delta}\right) \times \frac{1}{\delta} = -\frac{2U}{\delta}$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is negative the flow has separated. **Ans.**

Methods of Preventing Separation of Boundary Layer

1. Suction of the slow moving fluid by a suction slot.
2. Supplying additional energy from a blower.
3. Providing a bypass in the slotted wing.
4. Rotating boundary in the direction of flow.
5. Providing small divergence in a diffuser.
6. Providing guide-blades in a bend.
7. Providing a trip-wire ring in the laminar region for the flow over a sphere.

THANK YOU